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CALCULATIONS OF TEMPERATURES, STRESSES, AND STRAINS IN HEATING OF THERMALLY MASSIVE STEEL PRODUCTS

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Consideration is given to the methods of solution of problems of calculation of elastoplastic thermal stresses and strains which occur in heating of thermally massive bodies of a classical shape. The influence of thermophysical nonlinearities on the dynamics of the process of heating is analyzed.

In heating of thermally massive steel billets and ingots, the problem of calculation and analysis of the occurring (especially in the initial stage of the process) thermal stresses is topical and very significant. The nature of the appearance of these stresses depends on the scheme of heating, the frequency and amplitude of the heating sources, conditions of heat exchange with the ambient medium, the so-called fixity conditions, and other factors.

The stresses varying in a wide temperature interval can attain significant values and exceed the permissible ultimate strengths of specific grades of steel. Therefore, prediction of the possible values of the occurring temperature stresses enables one to evaluate the existing or designed regime of heating of a metal for correspondence to the criterion of adaptability to streamlined production and to realize different structural changes of the existing units for heating (heat treatment) of steel, etc.

The approaches to solution of the problems of thermoelasticity which are used by some authors are based on (approximate) engineering procedures of calculation of the temperature fields and rely on a number of simplifications required for analytical solution of such problems. The progress made in computer engineering makes it possible to conduct numerical experiments with a much higher accuracy of calculations and to evaluate the influence of one factor or another on the development of the process under study.

The problem of control of the heating of a metal with allowance for the maximum permissible temperature stresses assumes a successive solution of the problems of heat conduction and thermo-elastoplasticity.

The process of heating of thermally massive products by radiation and convection simultaneously in the case of a linear dependence of the thermophysical properties of a metal on the temperature is described by the system of partial differential equations in dimensionless form

$$(1 + e_c\Theta)\,\frac{\partial\Theta}{\partial \mathrm{Fo}} = \frac{1}{\xi^m}\,\frac{\partial}{\partial\xi}\left[\xi^m\,(1 + e_\lambda\,\Theta)\,\frac{\partial\Theta}{\partial\xi}\right], \quad \Theta\,(\xi, 0) = \Theta_0\,, \tag{1}$$

$$(1 + e_\lambda\,\Theta)\,\frac{\partial\Theta}{\partial\xi}\bigg|_{\xi=1} = \mathrm{Sk}\,[1 - \Theta_s^4\,(\mathrm{Fo})] + \mathrm{Bi}\,[1 - \Theta_s\,(\mathrm{Fo})]\,, \quad \frac{\partial\Theta}{\partial\xi}\bigg|_{\xi=0} = 0\,, \tag{2}$$

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where

$$e_c = \frac{\delta_c}{c_0} T_{\text{m}} \,; \quad e_\lambda = \frac{\delta_\lambda}{\lambda_0} T_{\text{m}} \,.$$

Thermally Massive Plates. Let us consider the solution of the problem of calculation of thermal stresses in an elastoplastic formulation for basic geometric shapes, i.e., a plate ($m = 0$) and a cylinder ($m = 1$).

Under combined loading, the deformation theory and the theory of flows [1] yield different results. However if the strains develop in one direction, results of calculations according to both theories of plasticity become closer.

For the plate under consideration, the coordinate plane *YOZ* is brought into coincidence with the middle plane of an unstrained plate while the *OX* axis is directed normally to the middle plane. The stresses σ_x, τ_{xy} and τ_{xz} can be disregarded [2].

According to the hypothesis of straight normals [2], the strain of each point of the plate can be represented as a sum of the strain of the middle surface and the flexural strain:

$$\varepsilon_y = \bar{\varepsilon}_y + x\chi_y \,, \quad \varepsilon_z = \bar{\varepsilon}_z + x\chi_z \,, \quad \gamma_{yz} = \bar{\gamma}_{yz} + x\chi_{yz} \,.$$

In the case under consideration, the plane is unfixed and the temperature varies just over the thickness. Therefore, we have

$$\sigma_y = \sigma_z = \sigma = \sigma\,(x)\,, \quad \tau_{yz} = 0\,, \quad \varepsilon_y = \varepsilon_z = \varepsilon = \varepsilon\,(x)\,, \quad \gamma_{yz} = 0\,.$$

We can write [3] that the force strain ε and the stresses σ are determined as

$$\varepsilon = B_1 + B_2 x - \alpha T\,, \quad \sigma = \frac{E}{1-\nu}\,(B_1 + B_2 x - \varepsilon^{\text{pl}} - \alpha T)\,,$$

where

$$B_1 = \int_{-h}^{h} \frac{E\alpha T}{1-\nu}\,(A_1 - A_2 x)\,dx + \int_{-h}^{h} \frac{E\varepsilon^{\text{pl}}}{1-\nu}\,(A_1 - A_2 x)\,dx\,; \quad B_2 = \int_{-h}^{h} \frac{E\alpha T}{1-\nu}\,(A_3 x - A_2)\,dx + \int_{-h}^{h} \frac{E\varepsilon^{\text{pl}}}{1-\nu}\,(A_3 x - A_2)\,dx\,;$$

$$A_1 = \frac{1}{\Delta}\int_{-h}^{h} \frac{Ex^2}{1-\nu}\,dx\,; \quad A_2 = \frac{1}{\Delta}\int_{-h}^{h} \frac{Ex}{1-\nu}\,dx\,; \quad A_3 = \frac{1}{\Delta}\int_{-h}^{h} \frac{E}{1-\nu}\,dx\,; \quad \Delta = \int_{-h}^{h} \frac{E}{1-\nu}\,dx \int_{-h}^{h} \frac{Ex^2}{1-\nu}\,dx - \left(\int_{-h}^{h} \frac{Ex}{1-\nu}\,dx\right)^2 .$$

Having introduced the notation

$$f\,(x) = \int_{-h}^{h} \frac{E\alpha T}{1-\nu}\,(A_1 - A_2 S - A_2 x + A_3 x S)\,dS - \alpha T\,,$$

$$K\,(x, S) = A_1 - A_2 S - A_2 x + A_3 x S\,, \quad \Phi\,(S, \varepsilon\,(S)) = \frac{E\varepsilon^{\text{pl}}}{1-\nu}\,,$$

we write for the force strain ε

$$\varepsilon(x) = \int_{-h}^{h} K(x, S)\, \Phi(S, \varepsilon)\, dS + f(x) \; . \tag{3}$$

Since the unknown strain appears in the function Φ, which in the general case is nonlinearly dependent on ε, the resultant equation (3) is the nonhomogeneous integral equation of Hammerstein [4], where $K(x, S)$ is the kernel of the equation.

Assuming that the material of the plate has linear hardening, we can express the plastic strain as follows:

$$\varepsilon^{\mathrm{pl}} = (1 - \Lambda)\,(\varepsilon - \operatorname{sign} \varepsilon\, \varepsilon_{\mathrm{yield}}) \; .$$

In this case, the nonhomogeneous Hammerstein equation becomes the Fredholm integral equation of the second kind [4]

$$\varepsilon(x) = f(x) + R \int_{-h}^{h} K(x, S)\, \varepsilon_0(S)\, dS$$

with the kernel

$$\overline{K}(x, S) = \frac{E}{1 - \nu}\,(1 - \Lambda)\, K(x, S) \; ,$$

where the parameter $K(x, S) = \dfrac{E}{1 - \nu}(1 - \Lambda)(A_1 - A_2 S - A_2 x + A_3 x S)$. The resultant integral equation of Fredholm is solved by the iteration method.

If we take $\varepsilon_0 = f(x)$ as the zero approximation, the iterative process converges when the kernel is quadratically summable:

$$\int_{-h}^{h} \int_{-h}^{h} \left| K(x, S) \right|^2 dx dS = B^2$$

and R satisfies the condition $|R| B < 1$.

The above algorithm of computation of stresses and strains has been evaluated using test examples. The case of bilateral cooling of a uniformly heated plate by a medium with an infinitely large coefficient of heat transfer was considered as a test. In [3], the results obtained according to the method presented above have been compared to the existing solutions [5, 6]. The error was no higher than 1%.

The body of mathematics presented enables one to analyze in detail the influence of thermophysical and physicomechanical parameters on the dynamics of the heating of a metal and the dynamics of the temperature stresses and also on the magnitude of the residual stresses.

In the numerical experiments, the temperature dependence of the parameters was assigned in the form of a table approximating this dependence by a piecewise-linear function. The changes in the thermophysical and physicomechanical parameters as functions of the temperature were taken from [7, 8]. In the case of a constant value of the parameter, its magnitude was determined as the mean-integral one for its variation in the interval of temperatures 273–1173 K.

The initial data for the numerical experiment were as follows: $2h = 0.27$ m, $T_0 = 293$ K, $T_{\mathrm{m}} = 1163$ K, $\alpha_{\mathrm{conv}} = 70$ W/(m^2·K), and $\sigma_{\mathrm{rad}} = 2.3 \cdot 10^{-8}$ W/(m^2·K^4); steel 45 was the metal used. As the mean-integral

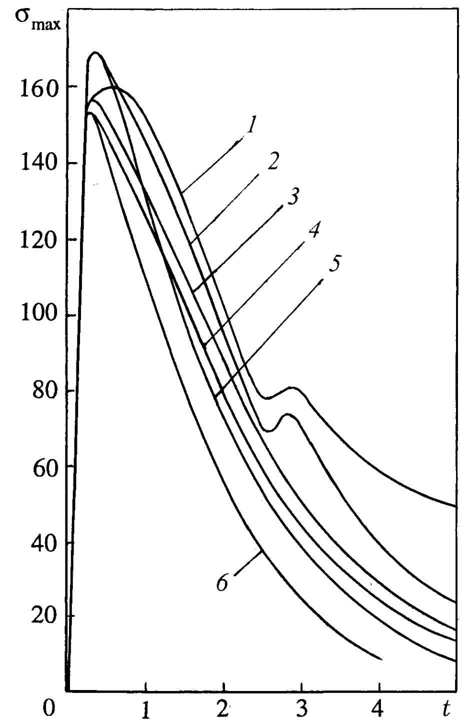

Fig. 1. Influence of the temperature dependence of thermophysical and physicomechanical parameters on the calculated maximum and residual stresses in a plate. σ_{max}, MPa; t, h.

values of the parameters, we determined $\overline{\lambda}$ = 45.6 W/(m·K), $\overline{E}$ = 250 MPa, $\overline{c}_v$ = 5.05 MJ/(m^3·K), $\overline{\sigma}_{yield}$ = 18.5·10^4 MPa, and $\overline{\alpha}$ = 13.05·10^{-6} 1/K.

Analysis of the results obtained shows that the heat capacity of the metal has the largest effect on the dynamics of heating. The selection of a constant value of the heat capacity reduces the duration of heating by almost 20%. Making such an assumption in calculations can lead to substantial errors in designing a heating device, in particular, to a disagreement between the calculated and actual output toward its reduction; therefore, it can prove a restrictive factor in the train "furnace–rolling mill."

Figure 1 shows the influence of the thermophysical and physicomechanical characteristics of a metal on the dynamics of the temperature stresses and on the magnitude of the residual stresses. It is clear that the values of the former are independent, in practice, of the method of assignment of thermomechanical parameters. At the same time, the values of the latter differ strongly by the end of the process. Taking the coefficient of linear expansion to be constant has the largest effect on the magnitude of the stresses. The adoption of constant values of the parameters can lead to an error of 3 to 5 times in evaluating the magnitude of the residual stresses toward their understatement and be the reason for premature failure of the rollers of the rolling mill; it can also lead to unreliable results in realization of high-speed regimes of heating, etc.

After checking the adequacy of the algorithm proposed, the authors performed a thermomechanical calculation of a steel plate in the case of its heating in a continuous furnace before rolling. The authors used the following initial data for the calculation: thickness 0.27 m and grade of the steel 45; the technological restrictions were as follows: surface temperature 1190^{o}C and temperature difference 25^{o}C; the regime of heating was three-stage.

The calculation results are given in Table 1. As follows from the data obtained, the massive plate attains the required temperature difference and surface temperature in 5.5 h; throughout the regular stage of heating, the temperature difference decreases. Analysis of the change in the temperature stresses shows that

TABLE 1. Change in the Temperatures T, the Elastic ε^{el} and Plastic Strains ε^{pl}, and the Stresses σ on the Surface and at the Center of a Massive Plate on Prolonged Heating before Rolling

t, sec	T, °C	ε^{el}	ε^{pl}	σ, MPa	Sign of load
360	195	–0.00122	–0.00071	–360.8	0
	20	0.00062	0	189.0	0
2160	451	–0.00097	–0.00079	–259.0	0
	320	0.00075	0	214.0	0
3960	659	–0.00054	–0.00315	–126.9	0
	527	0.00082	0.00013	207.9	0
5760	821	0.00030	–0.00266	60.5	2
	705	–0.00004	0.00030	–8.8	1
7560	921	0.00022	–0.00213	37.7	2
	828	–0.00010	0.00030	–20.5	1
9360	999	0.00026	–0.00185	39.2	2
	924	–0.00018	0.00030	–31.5	1
11160	1059	–0.00030	–0.00163	38.9	2
	998	–0.00026	0.00030	–38.0	1
12960	1106	0.00033	–0.00146	37.8	2
	1056	–0.00030	0.00029	–39.0	2
14760	1125	0.00034	–0.00128	37.1	2
	1086	–0.00031	0.00025	–38.4	2
16560	1154	0.00035	–0.00116	35.8	2
	1122	–0.00034	0.00024	–37.2	2
18360	1177	0.00037	–0.00107	34.6	2
	1151	–0.00035	0.00024	–36.0	2
20160	1190	0.00038	–0.00101	33.5	2
	1165	–0.00037	0.00022	–34.8	2

Note: 1) the upper line corresponds to the calculated data on the surface ($\rho = 1$), while the lower line corresponds to the data at the center of the plate ($\rho = 0$); 2) in the last column, 0 denotes load, 1 denotes unloading, and 2 denotes reverse flow.

they attain their maximum value (260.8 MPa) by the end of the inertial stage (360 sec). Thereafter the stresses gradually decrease.

Figure 2 gives the distribution of elastoplastic zones in the plate in prolonged heating. It is clear that in the initial period the elastoplastic strains develop from the heat-absorbing surface to the center of the plate. Next, on further heating, the zones develop from the periphery to the center and from the center to the periphery. The elastic zone decreases as the plastic strains manifest themselves. However, despite the significant surface temperature and temperature differences, the elastoplastic zone fails to cover the entire cross section of the plate by the end of the process of heating. This, in turn, can lead to the breakage of plane ingots and billets in subsequent rolling of them on blooming and plate mills and also to the deflections and buckling of the metal in heating.

Thermally Massive Cylinders. It proved difficult to obtain the analogous computational algorithms for determination of the fields of temperature stresses for a cylinder. Therefore, the authors used a somewhat different approach to the formulation and solution of the problem of thermoplasticity. In particular, the calculational relations were derived with allowance for the linear change in the thermophysical and physicomechanical characteristics of the cylinder material as a function of the temperature.

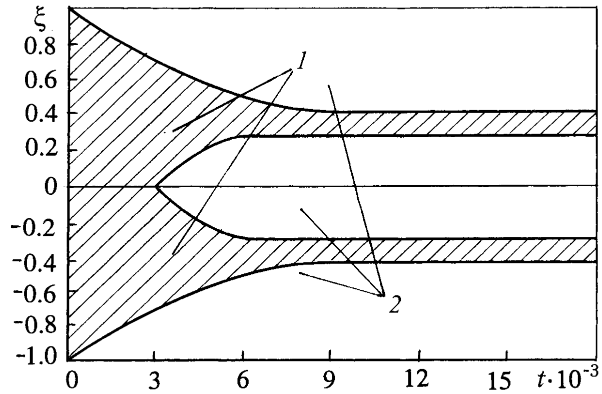

Fig. 2. Distribution of elastoplastic zones in a plate ingot in the case of prolonged heating: 1) zone of elastic strains; 2) zone of plastic strains. *t*, sec.

Since the solution of the problem of thermal stresses is aimed at performing calculations for thermal stability, we assume that in the inertial stage the values of the thermal stresses attain their maximum by the end of the inertial period (for the maximum values of the temperature differences). In selection of the yield conditions for steel, being guided by the recommendations of [9], we make the assumption of the incompressibility of the material in the zone of plastic strains. We assume that the zone of plastic strains appears on the peripheral layers of the cylinder, while in the axial zone the metal is ideally elastic.

For the convenience of mathematical transformations we will use dimensionless quantities. We assume that the elastic modulus, the coefficient of linear expansion, and the yield strength can be approximated by the linear dependences

$$E(\Theta) = E_0 (1 + e_E \Theta) , \quad \alpha(\Theta) = \alpha_0 (1 + e_\alpha \Theta) , \quad \sigma_{\text{yield}}(\Theta) = 1 - e_{\sigma_{\text{yield}}} \Theta_{\text{s}} ,$$

where

$$e_E = \frac{\delta_E}{E_0} T_{\text{m}} ; \quad e_\alpha = \frac{\delta_\alpha}{\alpha_0} T_{\text{m}} ; \quad e_{\sigma_{\text{yield}}} = \delta_{\sigma_{\text{yield}}} T_{\text{m}} .$$

It is necessary to note that in heating of massive ingots and billets the most dangerous are longitudinal tensile stresses on the axis of the body; in the case of the free ends of an infinitely long cylinder they will be written [10] as

$$\sigma_z = \sigma_r + \sigma_\theta .$$

Using the solution of the temperature problem (1) and (2) and omitting the intermediate mathematical computations [11], we write:

• the stresses in the zone of elastic strains

$$\sigma_r^{\text{el}} = C\left[1 + e_E(\Theta_{\text{s}} - A(1-\rho^2))\right]\left[\frac{e_\alpha A\Theta_{\text{s}} - e_\alpha A^2 + A}{4}(\rho_{\text{pl}\,0}^2 - \rho^2) + \frac{e_\alpha A^2}{12}(\rho_{\text{pl}\,0}^3 - \rho^3)\right] +$$
$$+ \ln \rho_{\text{pl}\,0}(-e_{\sigma_{\text{yield}}} A - 1 + e_{\sigma_{\text{yield}}} \Theta_{\text{s}}) - \frac{e_{\sigma_{\text{yield}}} A(1-\rho_{\text{pl}\,0}^2)}{2} , \tag{4}$$

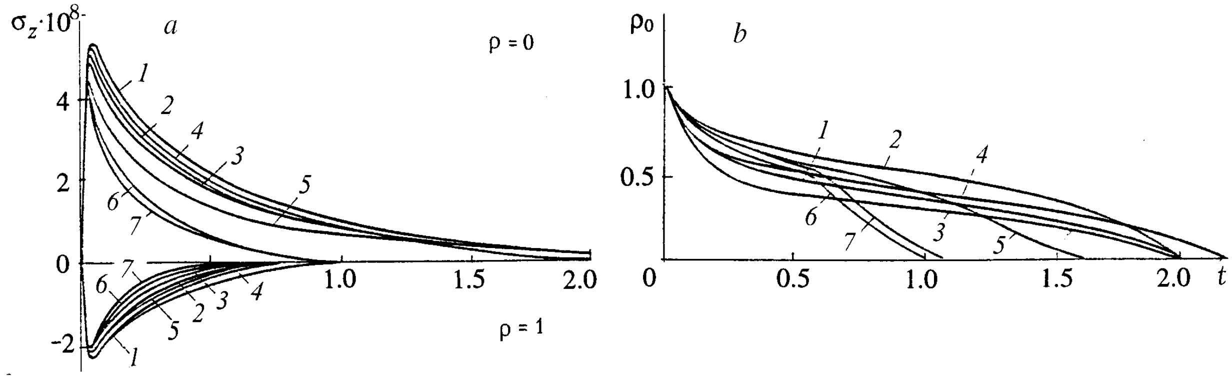

Fig. 3. Dynamics of thermal stresses (a) and the advance of the boundary of the plastic-strain zone ρ_0 (b) with time in a cylindrical ingot in relation to the selected variant of change of the thermophysical and physicomechanical properties of the metal: 1) $\lambda(T)$, $c(T)$, $E(T)$, and $\alpha(T)$; 2) $\lambda(T)$, $c(T)$, $E(T)$, and $\bar{\alpha}$; 3) $\lambda(T)$, $c(T)$, $\bar{E}$, and $\alpha(T)$; 4) $\lambda(T)$, $c(T)$, $\bar{E}$, and $\bar{\alpha}$; 5) λ, $c(T)$, $E(T)$, and $\alpha(T)$; 6) λ, $\bar{c}$, $E(T)$, and $\alpha(T)$; 7) λ, $\bar{c}$, $\bar{E}$, and α. σ_z, Pa; t, h.

$$\sigma_\theta^{\text{el}} = C\left[1 + e_E\left(\Theta_{\text{s}} - A\left(1-\rho^2\right)\right)\right]\left[\frac{e_\alpha A\Theta_{\text{s}} - e_\alpha A^2 + A}{4}\left(\rho_{\text{pl}\,0}^2 - 3\rho^2\right) + \frac{e_\alpha A^2}{12}\left(\rho_{\text{pl}\,0}^3 - \rho^3\right) - \frac{e_\alpha A^2}{4}\rho^4\right] +$$

$$+ \ln\rho_{\text{pl}\,0}\left(-e_{\sigma_{\text{yield}}}A - 1 - e_{\sigma_{\text{yield}}}\Theta_{\text{s}}\right) - \frac{e_{\sigma_{\text{yield}}}A\left(1-\rho_{\text{pl}\,0}^2\right)}{2}\,; \qquad (5)$$

• the stresses in the zone of plastic strains

$$\sigma_r^{\text{pl}} = \ln\rho\left(-e_{\sigma_{\text{yield}}}A + 1 + e_{\sigma_{\text{yield}}}\Theta_{\text{s}}\right) - \frac{e_{\sigma_{\text{yield}}}A\left(1-\rho^2\right)}{2}\,, \qquad (6)$$

$$\sigma_\theta^{\text{pl}} = \ln\rho\left(-e_{\sigma_{\text{yield}}}A + 1 - e_{\sigma_{\text{yield}}}\Theta_{\text{s}}\right) - \left(1 - e_{\sigma_{\text{yield}}}\Theta_{\text{s}}\right) - \frac{3e_{\sigma_{\text{yield}}}A\left(1-\rho^2\right)}{2}\,. \qquad (7)$$

The boundary of the plastic zone $\rho_{\text{pl}0}$ is found from the expression

$$C\left[1 + e_E\left(\Theta_{\text{s}} - A\left(1-\rho_{\text{pl}\,0}^2\right)\right)\right]\left[-\frac{e_\alpha A\Theta_{\text{s}} - e_\alpha A^2 + A}{2}\rho_{\text{pl}\,0}^2 + \frac{e_\alpha A^2}{6}\rho_{\text{pl}\,0}^3 - \frac{e_\alpha A^2}{2}\rho_{\text{pl}\,0}^4\right] =$$

$$= -\left[1 - e_{\sigma_{\text{yield}}}\left(\Theta_{\text{s}} - A\left(1-\rho_{\text{pl}\,0}^2\right)\right)\right]. \qquad (8)$$

In expressions (4)–(8), we have adopted the following notation: $A = \{\text{Sk}\,[1 - \Theta_{\text{s}}^4(\text{Fo})] + \text{Bi}\,[1 - \Theta_{\text{s}}(\text{Fo})]\}/\{2\,[1 + e_\lambda\Theta_{\text{s}}(\text{Fo})]\}$ in the regular stage, $A = \{\text{Sk} + \text{Bi}\,[1 - \Theta_0]\}/\{2\,[1 + e_\lambda\Theta_0] + \text{Bi}\}$ in the inertial stage, and $C = \{\alpha_0 E_0 T_{\text{m}}\}/\{(1-\nu)\sigma_{\text{yield}0}\}$.

Thus, we can propose the following algorithm of solution of the problem of thermomechanics:

(1) we determine the duration of the inertial period Fo_0 and the surface temperature $\Theta_{1\text{s}}^0$;

(2) we find the thermomechanical constants and the value of $\rho_{\text{pl}\,0}$ from formula (8);

(3) for $\rho \geq \rho_{pl\,0}$ we calculate the stresses in the zone of plastic strains from formulas (4) and (5) (if it exists) and the stresses in the zone of elastic strains (for $\rho < \rho_{pl\,0}$) from formulas (6) and (7);

(4) based on the solutions of Eqs. (1) and (2) we determine $\Theta_{2s}(\mathrm{Fo})$ at the instant of time Fo and, substituting its value into (4)–(8) in the order presented earlier, we find the distribution of the temperature stresses over the cross section of the body at this instant.

Just as for the case of a plate, the authors analyzed the influence of the thermophysical and physicomechanical properties of a metal on the values and development of elastoplastic stresses in heating of thermally massive bodies of a cylindrical shape.

Because of the fact that tensile stresses are the most dangerous from the viewpoint of the discontinuity of a material we confine ourselves to an analysis of the axial tensile stresses.

In the calculations, we took $2R = 0.23$ m, $T_0 = 20^{\circ}$C, $T_m = 1300^{\circ}$C, $\alpha_{conv} = 30$ W/(m^2·K), $\sigma_{rad} = 3 \cdot 10^{-8}$ W/(m^2·K^4), steel 45, $\lambda = 35$ W/(m·K), $c_v = 5$ MJ/(m^3·K), $E = 185$ MPa, and $\alpha = 15 \cdot 10^{-6}$ 1/K as the initial data.

The calculation results are presented in Fig. 3. As follows from the given plots, taking the elastic modulus to be a constant leads to a decrease of 6 to 7% in the level of stresses, which can create an unjustifiably overstated regime of heating in selecting the temperature regime of a heating device. The calculation according to other variants give values of the maximum stresses 6 to 20% lower on the average than calculation with variable parameters. The dynamics of change of the stresses and the boundary of the plastic zone substantially depends on the thermophysical properties and mainly on whether the heat capacity is assigned to be a constant or a variable.

Conclusions. The results obtained by the authors demonstrate that when the temperature regimes of heating of steel in industrial furnaces are selected according to the conditions of thermal stability it is necessary to calculate the occurring thermal stresses with allowance for the variability of thermophysical and physicomechanical properties. The developed methods of modeling of the heating of massive steel products in the shape of a plate and a cylinder according to the conditions of their thermal stability have been used in the calculations of industrial furnaces operating in the trains of rolling mills (tubular, plate, section, and axle) of metallurgical processes.

NOTATION

$\xi = x/R$, $\Theta(\xi, \mathrm{Fo}) = T(x, t)/T_m$, $\Theta_0 = T_0/T_m$, $\Theta_s = T_s/T_m$, and $\mathrm{Fo} = at/R^2$, dimensionless coordinate, running, initial, and surface temperatures, and time, respectively; a, thermal diffusivity of the body; R, characteristic dimension of the body; m, shape factor of the body; x, $T(x, t)$, T_0, T_s, T_m, and t, absolute coordinate (reckoned from the center), running, initial, surface, and heating-medium temperatures, and time respectively; $\mathrm{Bi} = \alpha_{conv}R/\lambda$ and $\mathrm{Sk} = \sigma_{rad}T_m^3R/\lambda$, Biot and Stark numbers respectively; α_{conv} and σ_{rad}, coefficients of heat transfer by convection and radiation; λ, thermal conductivity of the body; $\bar{\varepsilon}_y$, $\bar{\varepsilon}_z$, and $\bar{\gamma}_{yz}$, strains of the middle surface; χ_y, χ_z, and χ_{yz}, curvature that can be expressed in terms of the deflection of the plate u_x in the case of small deflections [2]: $\chi_y = \partial^2 u_x/\partial y^2$, $\chi_z = \partial^2 u_x/\partial z^2$, and $\chi_{yz} = \partial^2 u_x/\partial y \partial z$; ε, strain; σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{xz}, tensors of normal and tangential stresses in the plate; E, elastic modulus; α, coefficient of linear expansion; ε^{pl}, plastic strain; h, plate thickness; S, integration variable; ν, Poisson coefficient; $\Lambda = E_t/E$, dimensionless parameter of hardening; E_t, tangential modulus in the hardening region; ε_{yield}, strain corresponding to the yield strength; σ_{yield}, yield limit; E_0, α_0, σ_{yield0}, c_0, and λ_0, elastic modulus, coefficient of linear expansion, yield strength, heat capacity, and thermal conductivity at the initial temperature respectively; δ_E, δ_α, $\delta_{\sigma_{yield}}$, δ_c, and δ_λ, slopes of the straight lines E–T, α–T, σ_{yield}–T, c–T, and λ–T; σ_z, σ_r, and σ_θ, axial, radial, and tangential stresses for the cylinder; ρ, distance from the center of the cylinder; $\rho_{pl\,0}$, boundary of the plastic zone; c_v, heat capacity per unit volume; Fo_0, duration of the inertial period; Θ_{1s}^0, surface temperature at the end of the inertial period; $\Theta_{2s}(\mathrm{Fo})$, surface temperature at the instant of time Fo in a regular stage of heating.

Subscripts and superscripts: 0, initial; s, surface; m, medium; conv, convection; rad, radiation; t, tangential; x, y, z, coordinate axes; yield, yield; el, elastic; pl, plastic; max, maximum.

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